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Protational Monopoles, Non-Riemannian Geometry, and Quasars

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In pursuing analogies between gravitation and electrodynamics, the question occurs as to the possible existence of a gravitational analog of the (hypothetical) magnetic monopole. This entity cannot exist in standard general relativity, but could in a theory which makes use of non-Riemannian geometry. A theory involving such "protational monopoles" is formulated here, and comments are made on the possible relevance of such a theory for the spectral shifts and energy releases in such objects as quasars. But it is also pointed out that the quantization of mass necessitated by protational monopoles casts doubts upon their existence.

1. THE GRAVITATIONAL–ELECTROMAGNETIC ANALOGY

There are a number of interesting and suggestive analogies between gravitation and electrodynamics and, of course, some fundamental differences. Here I wish to consider one possible correspondence between these two classes of phenomena, the possible existence of a gravitational analog of the magnetic monopole.

It is well known that the mathematical formulation of the Newtonian theory of gravitation is very similar to that for electrostatics. The former theory can be summarized in the following set of equations:

$$m_i\ddot{\mathbf{r}} = m_g\mathbf{G}, \qquad \nabla\cdot\mathbf{G} = -4\pi k\rho, \qquad \nabla\times\mathbf{G} = 0 \tag{1}$$

The first equation of this set is that for the motion of a test particle with inertial mass m_i and passive gravitational mass m_g in a gravitational field $\mathbf{G}$. The other equations, in which k is the Newtonian gravitational constant and ρ the mass density, allow $\mathbf{G}$ to be calculated if sources and boundary conditions are given. The equations of electrostatics are obtained by changing m_g to q, the charge of the test particle, $\mathbf{G}$ to $\mathbf{E}$, the electric field, and k to a value depending on the system of electromagnetic units to be used. The actual correspondence between the two theories is, however, not quite as straightforward as such simple replacements might indicate. First, m_g/m_i is the same for all bodies (equivalence principle), while q/m_i is not. Secondly, only one sign of mass has been found in nature, while a body may have positive, negative, or zero charge. Thirdly, the constant k must be negative in the electromagnetic case: like masses attract, but like charges repel.

When we advance to Einstein's theory of gravitation, we find some other similarities between gravitation and electromagnetism. Throughout this section I shall assume that fields are weak, and that the speeds of test particles are small in comparison with that of light, which is now taken to be unity. In addition, it will be assumed that fields are stationary: $\partial g_{\mu\nu}/\partial t = 0$ in an appropriate reference frame, though the metric components g_{0i} need not vanish. (Greek indices range from 0 through 3, and Latin from 1 through 3.) The world line of a neutral test particle will be a geodesic of the curved space-time. With the stated assumptions, the geodesic equation yields, to first order in the speed of the particle (Weinberg 1972),

$$\frac{dv^i}{dt} \approx -\left\{ {}_0{}^i{}_0 \right\} - 2\left\{ {}_0{}^i{}_j \right\} v^j \tag{2}$$

The first term on the right of (2) corresponds to the Newtonian field G^i. In addition to this "gravitational field" **G** we shall also define a field **P** *via* $P^k = \varepsilon^{km}{}_n\{{}_0{}^n{}_m\}$, so that $\{{}_0{}^i{}_j\} = -\frac{1}{2}\varepsilon^i_{jk}P^k$. (We shall soon see that, in this situation, the Christoffel bracket of the first kind, $[0j, k]$, is antisymmetric in j and k.) Equation (2) can then be written

$$d\mathbf{v}/dt = \mathbf{G} + \mathbf{v}\times\mathbf{P} \tag{3}$$

The non-Newtonian force term involving the "protational" field **P** of Forward (1961; see also Penfield and Haus, 1967) corresponds to the Lorentz force on an electric charge in a magnetic field **B**. It is this $\mathbf{v}\times\mathbf{P}$ force which should produce the Lense–Thirring (1918) precession of the orbit of a satellite of a rotating planet.

We now require equations which will allow us to calculate the fields **G** and **P** from their sources. Since the $\{{}_\beta{}^\alpha{}_\gamma\}$ are the Christoffel affinities, our previous assumptions allow us to conclude that **G** is a gradient (of the Newtonian potential plus relativistic corrections), while, with the metric signature $-1, +1, +1, +1$,

$$[0j, i] \approx \tfrac{1}{2}(g_{0i,j} - g_{0j,i}) \tag{4}$$

P will thus be a curl, and we may write

$$\nabla \times \mathbf{G} = 0, \qquad \nabla \cdot \mathbf{P} = 0 \tag{5}$$

These relations, which are analogous to half of Maxwell's equations, follow from the Riemannian relation between metric and affinity, together with our slow-motion, weak-field approximations. Monopole sources for the protational field should, however, appear in this approximation if such entities exist.

The Einstein equations now give us the equations which correspond to the other half of the Maxwell set,

$$\nabla \cdot \mathbf{G} = -4\pi k \rho, \qquad \nabla \times \mathbf{P} = -16\pi k \rho \mathbf{U} \tag{6}$$

Here **U** is the velocity of the source material.

2. PROTATIONAL MONOPOLES

The possible existence of magnetic monopoles has received considerable attention since the early discussion of Dirac (1931). In the electromagnetic analogs of (5) and (6), source terms in the first set would apparently produce greater symmetry between the fields **E** and **B**. The existence of magnetic monopoles would also complicate matters somewhat, since the fields could not be written in terms of potentials in the usual way.

In view of the similarities between gravitation and electrodynamics which have been considered in the preceding section, it seems natural to ask if a gravitational analog of the hypothetical magnetic monopole might exist: our previous terminology suggests the name "protational monopole" for such an entity.

The simplest way to introduce protational monopoles would be by means of equations (5) and (6), with neglect of their origin in general relativity. Such a phenomenological theory will be used in the last section, but, because of the great success and aesthetic appeal of general relativity, an attempt should be made to follow the route marked out by that theory. The existence of protational monopoles would require the introduction of source terms in the set (5). Since this set of equations follows, with our approximations, from the basic relation between metric and affinity of Riemannian geometry, the existence of a static protational monopole will require the use of non-Riemannian geometry for its description.

A great deal of work has been done on non-Riemannian unified field theories during the past 65 years. I shall assume here that the equation of motion of a test particle is given by the path equation

$$dU^\sigma/ds + \Gamma^\sigma_{\alpha\beta} U^\alpha U^\beta = 0 \tag{7}$$

with the affinity given by

$$\Gamma^\sigma_{\alpha\beta} = \{_\alpha{}^\sigma{}_\beta\} + S_{\alpha\beta}{}^\sigma \tag{8}$$

The choice (7) is, of course, not a unique possibility. Since an antisymmetric part of $S_{\alpha\beta}{}^\sigma$ would make no direct contribution to the path equation, we may assume that this tensor is symmetric in α and β.

If we follow the lines of our previous argument, we will now obtain an addition $\mathbf{P}'$ to the protational field as $P'^k = \varepsilon^{km}{}_n S_{0m}{}^n$, together with other terms in the equation of motion for a test particle. Since there will be, in general, no requirement that $\mathbf{P}'$ have a vanishing divergence, there can be protational monopoles.

As a specific example, we may consider the case of Weyl's (1918) geometry, in which $S_{\alpha\beta}{}^\sigma$ is given in terms of a vector field κ^α by

$$S_{\alpha\beta}{}^\sigma = g_{\alpha\beta}\kappa^\sigma - g_\alpha{}^\sigma\kappa_\beta - g_\beta{}^\sigma\kappa_\alpha \tag{9}$$

(This follows Eddington, 1923, with slightly clearer positioning of indices.) The path equation then gives the following terms in the three-acceleration in addition to those which one obtains in general relativity:

$$\mathbf{a}' = -g_{00}\boldsymbol{\kappa} + \mathbf{v}\times(2\mathbf{g}\times\boldsymbol{\kappa}) - 2[\mathbf{g}(\mathbf{v}\cdot\boldsymbol{\kappa}) - \mathbf{v}\kappa_0] \tag{10}$$

Here $\boldsymbol{\kappa}$ is the spatial part of κ^α and $\mathbf{g}$ is a "vector" made up of the metric components g_{0i}. The earlier approximations are still made here, but no assumptions have been made about the magnitude of the new vector field. We still have $U^0 \approx 1$.

The first term in (10), $\mathbf{G}' = -g_{00}\boldsymbol{\kappa}$, is an addition to the "gravitational" field, while $\mathbf{P}' = 2\mathbf{g}\times\boldsymbol{\kappa}$ is the addition to the protational field. [In addition, we must note the final "resistive" term in equation (10).] In general, $\nabla\cdot\mathbf{P}' \neq 0$ and $\nabla\times\mathbf{G}' \neq 0$, so that $\mathbf{P}'$ and $\mathbf{G}'$ are fields which may be thought of as arising from the protational sources associated with the field $\boldsymbol{\kappa}$.

Weyl's geometry has been considered here only in order to provide a simple example of the type of non-Riemannian geometry in which we are interested. Even within the context of Weyl's theory there are other possible approaches—see, e.g., Maeder (1978).

We should also note here the discussions of "gravitational dual charge" by, e.g., Lubkin (1977). While our "protational monopole" could exist in an asymptotically flat space-time, Lubkin shows that one cannot surround a gravitational dual charge with a two-dimensional bag which is everywhere spacelike. Thus these two theoretical entities are different, though sharing related motivations.

3. POLE STRENGTH, SPECTRAL SHIFTS, AND QUASARS

Weyl introduced the vector field κ^{μ} in the above manner in an attempt to unify gravitational and electromagnetic interactions: κ^{μ} was supposed to represent the electromagnetic four-vector potential. This attempt is generally considered to have been unsuccessful, largely because of the nonintegrability of length produced by electromagnetic potentials, leading to such effects as changes in the characteristic frequencies radiated by atoms which could hardly have escaped detection. (Einstein, 1918). But it is possible that κ^{μ} could have some quite different meaning. This vector field could be negligible within even our local group of galaxies, but have significant values in other parts of the universe. If this were the case, those regions would have properties quite different from our region. The dimensions and characteristic frequencies of atomic systems would be changed (the increment in a magnitude L of a vector on transport through a displacement dx^{α} being given by $dL/L=\kappa_{\alpha}dx^{\alpha}$). Thus nonnegligible values of κ^{α} in some region of the universe would mean that the radiation emitted from that region would show systematic spectral shifts.

In order to carry the discussion further, it is necessary to have an estimate of the protational pole strength associated with the new field, and for this purpose it will be simplest to consider a quasi-Newtonian theory of gravitation with protational sources. Just as for the magnetic monopole, we can obtain the desired relation from the requirement that the angular momentum associated with a protational monopole and an ordinary mass be properly quantized. We may consider the case in which the monopole, with pole strength p, is at rest, and a particle of mass m moves in its protational field $\mathbf{P}=-k\mathbf{r}/r^{3}$. With the speed of light restored in our formulas, the rate of change of orbital angular momentum $\mathbf{L}$ will be given by

$$d\mathbf{L}/dt=\mathbf{r}\times m\,d\mathbf{v}/dt=-(kmp/c)\mathbf{r}\times(\mathbf{v}\times\mathbf{r}/r^{3})=-(kmp/c)\,d(\mathbf{r}/r)/dt$$

Thus $\mathbf{L}+kmp\mathbf{r}/cr$ is constant. kmp/c is the magnitude of the intrinsic field momentum, whose minimum value will be $\hbar/2$. Thus we obtain

$$p=\hbar c/2km \tag{11}$$

for the minimum value of the protational pole strength. Mass does not appear to be quantized in the same way that charge is, a serious difficulty which will be discussed in the next section. For the present, we may note that if we let m be the electron rest mass, the smallest rest mass which is known to occur in nature for a nonrelativistic particle, the corresponding value for the minimum protational pole strength is 3×10^{17} g.

This is a huge mass for an elementary particle, and a relatively small number of such particles could produce significant gravitational effects. (Of course we must remember that this represents a novel type of "mass" as well.) Because of numerical coincidences, the mass obtained here is not far from that which is significant for the evaporation of black holes *via* the Hawking (1975) process.

Significant numbers of protational monopoles in a region of space would be expected then to produce large spectral shifts, and also the release of great amounts of energy. These features remind one of some of the most prominent features of quasars. It is at least conceivable that some of the still-puzzling effects associated with such objects might be due to the presence of protational monopoles.

4. PROTATIONAL MONOPOLES AND QUANTIZATION OF MASS

Protational monopoles are perhaps interesting theoretical entities, but the quantization of rest mass required by equation (11) seems to be a quite serious argument against their real existence. It is not impossible that rest mass is quantized, though the quantum would, of course, have to be much smaller than the electron mass used in the previous section for illustrative purposes. A "graininess" on the scale of, for example, the present limits on the photon rest mass (see, e.g., Murphy and Burman, 1978) could easily have escaped detection.

On the other hand, it must be admitted that there is certainly no evidence for a quantization of rest mass of this sort, and such a phenomenon would require major modifications in physical theories which are well established. If such quantization is rejected, then there appears to be no place for protational monopoles.

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